# **EVOLUTIONARY MODELLING IN DISCRETE MULTICRITERIA DECISION AIDING: DETERMINING THE PARAMETERS OF ELECTRE METHODS**

Carlos Francisco Simões Gomes Universidade Federal Fluminense & Ibmec-RJ cgomes@ibmecrj.br

> Luiz Flavio Autran Monteiro Gomes Ibmec-RJ <u>autran@ibmecrj.br</u>

> > Solange Fortuna Lucas Ibmec-RJ & IBGE solange.lucas@ibge.gov.br

**Abstract.** This article uses the concepts of Genetic Algorithms and Transgenetic Algorithms associated to a Multicriteria Decision Support System for discrete alternatives. The aim of this research is defining an important parameter of ranking methods, in this case, the preference threshold, p. This is one of the crucial parameters for the application of multicriteria methods of the French School of Multicriteria Decision Aiding that are embedded in that system. The use of Genetic Algorithms was shown to be a very useful tool for helping decision analysts in the choice of the thresholds values needed for using these methods.

**Key-words:** Multicriteria Decision Support System – Genetic Algorithms – Multiobjective Evolutionary Algorithm

**Resumo.** Este artigo faz uso dos conceitos de Algoritmos Genéticos e Algoritmos Trasngenéticos associados a um Sistema de Apoio Multicritério à Decisão e aplicados à análise de alternativas discretas. O objetivo principal da pesquisa aqui relatada é a definição do limite de preferência p, um importante parâmetro de métodos de ordenação. Esse parâmetro é um elemento indispensável para aplicação de métodos multicritério da Escola Francesa do Apoio Multicritério à Decisão embutidos naquele sistema. Verifica-se que o emprego de Algoritmos Genéticos é uma ferramenta muito útil para ajudar analistas na seleção dos valores limites necessários à aplicação desses métodos.

**Palavras-chaves:** Sistemas de Apoio Multicritério à Decisão – Algoritmos Genéticos – Algoritmo Evolutivo Multiobjetivo

## 1. Introduction

Genetic Algorithms (GAs) are metaheuristic, inspired by evolution and natural selection. GAs were introduced by John Holland in 1960 and are computational research methods based on genetic mechanism and natural evolution. In this way, they are used to solve search and optimization of adaptive form problems, based on the genetic process and the evolutionary process of living organisms. Populations composed of these organisms evolve according to the principles of natural selection and the survival of the fittest. GAs are generally efficient in the search for "good" solutions, according to the intervals or parameters considered. One of the advantages of their use is the simplification of the formulation and solution of optimization problems [1], [2], [3], [4].

Transgenetic Algorithms (TAs), in turn, are evolutionary algorithms which base their metaphor on a symbiogenetic process. Symbiogenetics is an evolutionary theory where individuals of different natures – that is, of different species – unite to form a new individual. Symbiogenetics places greater emphasis on the positive effects resulting from genetic interrelationships between individuals of different species rather than selection via reproduction of the fittest. Symbiogenetic relationships are more easily identified in the context of microorganisms, but may also be the explanation for the greatest innovations of life, such as the eukaryotic cell and cellular metabolism [5].

TAs use information from diverse sources and their process is strongly dependent on the quality of this information. It is also important for the information to be renewed during the procedure so as to prevent the premature convergence of the algorithm [6].

The symbiogenetic process of TAs is developed by the interaction of two populations of individuals of distinct species: chromosomes and transgenetic vectors. The interaction of the populations results in the forming of individuals fitter for environmental survival. In the evolutionary process, information is obtained both from the environment and from the evolutionary state of the populations. The interaction of the species occurs at three levels:

• The first contains the population of chromosomes which represent the memory of the result of the evolutionary process, in other words, the algorithmic search.

• The second level is composed of transgenetic vectors responsible for promoting the intensification and diversification of the population of chromosomes.

• The third level is formed by the rules which administrate the process of interaction between the populations of vectors and chromosomes, in this way characterizing a specific evolutionary scenario [7].

The symbiogenetic paradigm suggests that genetic or non-genetic information obtained a priori at the beginning of the evolutionary process can be used to guide the evolution, as the evolutionary past of the individuals is essential to enable evolutionary leaps. It is precisely in the union of the information contained in the individuals that the power for evolutionary acceleration resides. On the other hand, because it deals with an evolutionary process, information obtained during the process can also be used [8].

As the transgenetic process belongs to the dimension of relationships between species, the exchange of information between individuals of the same species is an accessory element which can, in addition, simply not exist. In this way, the evolution of the population of chromosomes can occur without there being a direct exchange of information between their individuals. This exchange can be reflected in crossings or in the occurrence of mutations, a typical effect of the autonomous transformation of a population. The level associated with the rules for administrating the transgenetic model corresponds to the project and coordination of a process of inter-relationships between populations of individuals of different natures. However, one point is fundamental here: the species obtain and share information which increases their chances of survival, even though this in fact implies their radical transformation, leading to the creation of a new species [9].

For symbiogenetics to function it is necessary for the species to be able to interact at a genetic level, even though they are of different species. In addition to this, each species must possess useful and complementary information. In the computational approach, the viability of genetic interaction is trivially guaranteed by the possibility of altering the configuration of the chromosomes and transgenetic vectors. On the other hand, it becomes essential for there

to be compatibility among these interactions and that they have a common direction in the search for chromosomes [10].

This article uses the concepts of GAs and TAs associated to a Multicriteria Decision Support System (MDSS), for discrete alternatives, with the purpose of defining an important ranking parameter, in this case, the preference threshold p. This is one of the crucial parameters for the application of multicriteria methods of the French School of Multicriteria Decision Aiding [11], [12], which are inserted in the MDSS in question.

## 2. The Multicriteria DSS

The MDSS used in this study was THOR (an acronym for AlgoriTmo Híbrido de ApoiO MulticritéRio à Decisão para Processos Decisórios com Alternativas Discretas, in Portuguese, or Multicriteria Decision Support Hybrid Algorithm for Decision Making Processes with Discrete Alternatives) which simultaneously aggregates the concepts of Rough Set Theory, Fuzzy Set Theory and Preference Theory. THOR is a Decision Support System for the multicriteria ranking of discrete alternatives, which eliminates redundant criteria simultaneously considering if the information is dubious - when using Rough Set Theory and if there is an increase in imprecision in the decision process - in which case Fuzzy Set Theory is used. In this way, imprecision is quantified, using it in the multicriteria decision support process. The concept of quantifying the imprecision associated with the weights and the classifications of the alternatives, put into operation in THOR, arises from the fact that the judgment values, because of their inherent subjectivity, cannot always be expressed in secure and precise ways. When using THOR, the simultaneous input of data into the process from multiple decision makers is also permitted, enabling these to express their judgment values in scales of ratios, intervals or ordinals, in addition to the execution of the decision making process without necessarily attributing weights to the criteria [13].

The analytical modelling embedded in THOR is based on the ELECTRE methods of the French School of Multicriteria Decision Aiding [14], [12], [15]. In this way, the following additional elements may be necessary for the application of THOR:

(i) A weight for each criterion, representing the relative importance among them;

(ii) A preference threshold (p) and another for indifference (q) for each criterion;

(iii) Discordance;

(iv) Pertinence of the values of the weights attributed to the criterion, as well as the pertinence of the classification of the alternative in the criterion.

Following the terminology of the French School, with g(.) being a decision criterion and a and b any two alternatives, the indifference threshold q is expressed by a function q[g(a)], which can be constant in some situations, and represents an upper limit (q) for the difference g(b)-g(a); any value for this difference less than q is not sufficient to guarantee strict preference, or even weak preference of b over a. The preference threshold, in turn, is represented by a function p[g(a)], which represents the difference g(b)-g(a); it can be constant in some situations and represents a lower limit, below which there is not sufficient information to opt for a strict preference of b over a. Continuing to follow the terminology of the French School, discordance consists of the fact that there are no criteria in which the intensity of preference of b in relation to a goes beyond an acceptable limit [16], [17].

It should be stressed that the relationships of outranking reached by means of THOR, instead of only indicating dominance, have a numerical quantitative which represents the "value of the alternative", through an additive value function. This approximation permits the dominance relationship and the hierarchy of the values of the alternatives to be represented. Three situations are admitted for one alternative to be better than another:

п

g

j

S

In those last three expressions the relationships of strict preference P, weak preference Q and indifference I are characterized in the formula below:

Situation S1, only takes into account the alternatives a for which aPb, with b being any other alternative. In this way, comparing a with b, we can identify the criteria in which aPb, taking into consideration the thresholds of preference, indifference and discordance, checking if the condition imposed is satisfied. If satisfied, we know that a dominates b. Afterwards, the criteria weights in which this condition was met are added. For another alternative c, the same procedure described previously is repeated. The final scoring of alternative a will be the sum of the values obtained. For the situation S2, the alternatives for which aPb and aQb are taken into account. In situation S3, the alternatives for which aPb, aQb and alb are taken into account. It should be noted that the last two situations (S2 and S3) are less rigorous than the first (S1), so that a smaller difference permits one alternative to be classified better than another [12], [18].

The detailed description of the THOR algorithm and the software which supports it are in [13].

## 3. Evolutionary Modelling for Determining p

The dynamic of evolutionary modelling is relatively simple. Once an initial population is formed, the operations of crossing and mutation in individuals are carried out, prioritizing in some way the best adapted individuals, until a stop criterion is reached; at the end of the process, it is expected that the individuals have evolved to a good solution. It is important to analyze in what way some parameters influence the behaviour of the GAs so that it is possible to establish them according to the necessities of the problem and the resources available.

The application of Evolutionary Modelling for determining p was part of a non-published evaluation study of Brazilian cities. A team of experts selected around 5,600 Brazilian cities whose basic data are available from the Brazilian Institute for geography and Statistics. Ten evaluation criteria were considered in that study. At first all criteria were dealt with as true criteria, in the terminology of the ELECTRE methods, making use of p and q as equal to zero. Later on p and q were made equal to ten percent of the average scores of the cities according to the criteria. With this, p and q became indeed quasi-criteria [12]. By using Evolutionary Modelling we were able to:

- (a) Find the value of p and q for the present statistical population of the cities
- (b) Determine values for p and q for future statistical populations.
- (c) It is understood that cities function as populations treated by GA because:
- (d) Cities evolve, improve or become worse along their existences.
- (e) Cities can have parts that evolve just like a cell division, i.e. a city may split in two other cities, although the same chromosomes are not kept. Those chromosomes are the scores of the cities according the each criterion, thus generating a mutation.

The above reasons supported the use of Evolutionary Modelling for determining p.

560 cities were randomly selected from the original set of around 5,600 cities and this generated a 560 by 10 matrix. In order to rank these 560 cities the THOR MDSS was utilized. There follow some comments on the principal parameters of the process:

a) Size of the Population: the size of the population affects the global performance and the efficiency of the GAs. A "small" population offers a smaller coverage of the search space, causing a fall in performance. A "large" population supplies a better coverage of the domain of the problem and prevents premature convergence for local solutions. However, with a large population greater computational resources become necessary or a longer time processing the problem. In the face of this consideration, in the study approached in this article, an initial study population (sampling) was used with 560 alternatives (10% of the total alternatives) of a population of 5600 (this was the first generation). Afterwards, in the second generation, 1/3 of the alternatives coming from the total population of 5600; another 1/3 of the original sample was "genetically" altered, this alteration occurred by alteration

in the parameters of some alternatives (mutation) or junction of the parameters of the alternatives removed, creating new alternatives (crossing); and another third was not altered. In the third generation the best classified half in ranking was chosen from the first generation and the best classified half from the second generation (concept of elitism).

b) Crossing Rate: the larger the rate, the faster new structures are introduced into the population.

c) Mutation Rate: a low rate of mutation prevents a given position from stagnating at a value, as well as enabling any point to be reached in the search space. With a very high rate, the search becomes essentially random. The concepts of mutation and crossing were used together in the second generation when 1/3 of the alternatives initially used were altered.

d) Generation Interval: controls the percentage of the population which will be substituted during the next generation. Two generation intervals were considered, as explained previously.

e) Substitution of the Population: The majority of genetic algorithms make a generational type of substitution of the population, that is, all the elements of the current population are substituted by descendants. In some applications, the concept of elitism is used (applied in the third generation), that is, some of the solution proposals are preserved, those of the best quality, especially the incumbent. In the study presented here, the strategy of generational substitution of the population was not used. In this way, the proposal consisted of substituting some alternatives, creating mutations, making crossings, altering the population and conducting a new analysis, using the concept of elitism.

## 4. Genetic Operators Used

The genetic operators transform the population through successive generations, extending the search until a satisfactory result is reached. For this purpose, it is necessary for the population to diversify and maintain adaptation characteristics acquired by previous generations. In this study only one generation was introduced.

The mutation operators are necessary for introducing and maintaining the genetic diversity of the population, arbitrarily altering one or more components of a chosen structure, in this way, supplying means for new elements to be introduced into the population. Mutation assures that the probability of reaching any point in the search space will never be zero. The mutation operator is thus applied to the individuals which, in this study, were alternatives. This application can be done in the following two ways:

(i) Multi-point: is a generalization of the idea of exchanging genetic material through points, where as many crossing points as desired can be used.

(ii) Uniform: does not use crossing points, but determines, through a global parameter, the probability of each variable being exchanged between the parents.

A multi-point concept was chosen for this study due to the fact that it is an explanatory study/work and consequently there is some difficulty in establishing the global parameter. The points used here were the classifications of the alternatives according to the multiple criteria, with the total number of these equal to 10 criteria.

Regarding the programming of genetic algorithms, there is an important aspect which differs a little from the habitual programming: it is more difficult to detect errors in the programming as the various lines of code will represent improvements on the initial scheme of evolution and, in many cases, the program will function and still supply apparently good results despite executing incorrect functions or ones that at least do not do what is expected of them. In the present study, though, particular attention was paid to this aspect, following the orientation of similar studies [19].

## 5. Results from the Proposed Model

Initially the total sampling of the alternatives in the criteria was checked, calculating the average of the classifications of the alternatives per criterion, afterwards calculating the standard deviation. The value of p was estimated as being 10% of the value of the standard deviation. The value of p was determined as being 10% of the standard deviation. In this way, Table I was obtained:

The value of p was determined as follows:

1) The average value was computed for the 10 criteria and for the 560 cities.

2) Next, the standard deviations for each average and for the 10 criteria over the 560 cities were determined.

3) Finally, it was established that p would be equal to ten percent of the standard deviation of the average for each criterion.

It was proposed initially to use different values for p and q. However, computational results generated values of p and q with differences below tem percent and this supported the use of equal values for p and q.

Designation of the	Criteria weights	Values of p
Criteria		
С	0.268618107	0.024
D	0.020431915	0.024
E	0.028089333	0.025
F	0.036145955	0.026
G	0.044772867	0.024
Н	0.054255214	0.024
Ι	0.065106258	0.029
J	0.078352518	0.02
K	0.096389212	0.022
1	0.126402006	0.025

Table I – Criteria, weights and values of p

After this a sampling was made, composed of 560 alternatives from a total population of 5,600 alternatives, the ranking of the 560 alternatives was obtained using the THOR Decision Support System. The alternatives were introduced through loading an Excel<sup>®</sup> spreadsheet. Proceeding in this way, the suggestions for the values of p were generated as shown in Table II. The first generation values were compared with the original values of p. The values suggested did not alter the original ranking of the alternatives.

Original values	Values suggested in the first	Variations
of p	generation	
0.024	0.097666	306.94%
0.024	0.097666	306.94%
0.025	0.098666	294.66%
0.026	0.099666	283.33%
0.024	0.097666	306.94%
0.024	0.097666	306.94%
0,029	0.102666	254.02%
0,02	0.093666	368.33%
0,022	0.095666	334.85%
0,025	0.098666	294.66%

**Table II** – Comparison between the original values and the values suggested for p, in the first generation

It can be observed, from Table II, that the values of p were chosen in a very conservative way. Next, the alteration in the sampling was made as previously explained and the new values presented in the first column of Table III were obtained:

First generation	Second generation	Variations
0.097666	0.140623	43.98%
0.097666	0.140614	43.97%
0.098666	0.140622	42.52%
0.099666	0.140524	40.99%
0.097666	0.140724	44.09%
0.097666	0.140124	43.47%
0.102666	0.140122	36.48%
0.093666	0.140123	49.60%
0.095666	0.140524	46.89%
0.098666	0.140024	41.92%

Table III – Comparison between the second and first generations of the values of p

Analyzing Table III, it could be seen that the second generation had a comparatively smaller variation in relation to the first generation than the first generation had in relation to the initial values. It was decided to make a third generation, using the best classified half of each sample. Great care was taken not to repeat alternatives so that the study could continue with 560 alternatives. It was then observed that only 4 alternatives remained in the two samplings in the interval characterized as 50% higher, of each sampling, as shown in Table IV. The values obtained from the second generation were chosen for the value of p.

Table IV – Comparison of the second and third generations of the values of p

Third	Second Generation	Variations
Generation		
0.150508797	0.140623	7.03%
0.152425576	0.140614	8.40%
0.150535851	0.140622	7.05%
0.150459047	0.140524	7.07%

0.150616897	0.140724	7.03%
0.149974717	0.140124	7.03%
0.149426101	0.140122	6.64%
0.150450065	0.140123	7.37%
0.153030636	0.140524	8.90%
0.149755668	0.140024	6.95%

Alterations less than 8.91% were observed from the third to the second generation alterations. As a result, it was considered that there had been a convergence of the values.

## 6. Conclusion

The use of genetic algorithms has been shown to be important sources in helping decision analysts choose the values of the thresholds of transitivity. The analysis presented here suggested that, for the application of the THOR Decision Support System, the values of p and q is equal: the study started with pseudo-criteria, where p is equal to 10% of the standard deviation and 10% greater than q, both being different to zero.

The use of GAs assisted the decision agent in solving two problems in the selection of p:

a) a value which is suitable to the study

b) a value which remains suitable for future studies, for the same problem.

The suggestion was that quasi-criteria be adopted, with p = q and with these values greater than the original ones. It could be seen that the initial values for p and that were set were indeed conservative values. As a matter of fact one could consider that there was a strong preference for one city against other cities and according to a given criterion. With the values for p and q that could indeed become an indifference. This has shown that a city may need a larger increase in criteria weighting in order to lead to a more realistic scoring when compared against another city.

For future research it is expected that similar approaches based on Evolutionary Modelling can be developed for other multicriteria methods, leading to processes of parameter generation more based on learning than on subjectivity.

#### Acknowledgement

Research leading to this article was partially supported by the National Council for Scientific and Technological Development (CNPq) of Brazil through Process No. 310603/2009-9.

#### 7. References

1. Gandibleux, X.; Ehrgott, M. "1984-2004 – 20 Years of Multiobjective Metaheuristics. But What About the Solution of Combinatorial Problems with Multiple Objectives?" In: Coello Coello et al. (Eds): *Evolutionary Multi-criterion Optimization EMO 2005 LNCS 3410*, p. 33-46, Berlin: Springer-Verlag (2005).

2. Quintão, F. P.; Nakamura, F. G.; Mateus, G. R., "Evolutive Algorithm for the Dynamic Covering Problem Applied to the Management of Wireless Sensors Network", SBPO XXXVII, Gramado: Brazilian Society of Operations Research (2005).

3. Alencar, M. P. de L.; Villareal, F.; Romero, R. "A Specialised Genetic Algorithm for the Generalized Assignment Problem", SBPO XXXVII, Gramado: Brazilian Society of Operations Research (2005).

4. Ramos, I. C. de O.; Goldbarg, M. C.; Goldbarg, E. F. G.; Dória Neto, A. D.; Farias, J. P. F. "A Statistical Technique for the Solution of the travelling Salesman Problem via an Evolutionary Algorithm", SBPO XXXVII, Gramado: Brazilian Society of Operations Research (2005).

5. Michalewicz, Z. Genetic Algorithms + Data Structures = Evolution Programs, Berlin: Springer-Verlag (1996).

6. Van Veldhuizen, D.A.; Lamont, G.A. "Multiobjective Evolutionary Algorithms: Analysing the State-of-the-Art", *Evolutionary Computation*, 8, p. 1-26 (2000).

7. Zitzler, E.; Deb, K. "Comparison of Multiobjective Evolutionary Algorithms: Empirical Results", *Evolutionary Computation*, 8 (2), p. 125-148 (2000).

8. Bäck, T. (ed.) *Evolutionary Algorithms in Theory and Practice*, New York: Oxford University Press (1996).

9. Coello Coello, C.A. "A Comprehensive Survey of Evolutionary-based Multiobjective Optimization Techniques", *Knowledge and Information Systems*, 1(3), p. 269-308 (1999).

10. Deb, K. Multi-objective Optimization using Evolutionary Algorithms, Chichester: Wiley (2001).

11. Roy, B. *The European School of MCDA: Emergence, Basic Features and Current Works*, Paris: Laboratoire d'Analyse et Modélisation de Systemes de Aide à la Décision/Université Paris Dauphine (1995).

12. Roy, B.; Bouyssou, D. Aide Multicritère à la Décision Méthodes et Cas, Paris : Economica (1993).

13. Gomes, C. F. S. *THOR A Hybrid Algorithm for Discrete Multicriteria Decision Aiding*, Doctoral Thesis in Production Engineering, COPPE/UFRJ, Rio de Janeiro (1999).

14. Roy, B. *The Outranking Approach and the Foundations of ELECTRE Methods*, Paris: Laboratoire d'Analyse et Modélisation de Systemes de Aide à la Décision/Université Paris Dauphine (1989).

15. Belton, V.; Stewart, T. J. *Multiple Criteria Decision Analysis An Integrated Approach*, Boston: Kluwer Academic Publishers (2002).

16. Vanderpooten, D. "The Construction of Prescriptions in Outranking Methods", In: Bana e Costa, C.A. (ed.): *Readings in Multiple Criteria Decision Aid*, p. 184-215, Berlin: Springer-Verlag (1990).

17. Roy, B. *Multicriteria Methodology for Decision Aiding*, Boston: Kluwer Academic Publishers (1996).

18. Triantaphyllou, E. *Multicriteria Decision Making Methods: A Comparative Study*, Boston: Kluwer Academic Publishers (2000).

19. Leyva-Lopez, J. C.; Aguilera-Contreras, M. A. "A Multiobjective Evolutionary Algorithm for Deriving Final Ranking from a Fuzzy Outranking Relation", In: Coello Coello et al. (Eds): *Evolutionary Multi-criterion Optimization EMO 2005 LNCS 3410*, p. 235-249, Berlin: Springer-Verlag (2005).