COMPARING TWO EVALUATIONS OF RESIDENTIAL PROPERTIES: MULTICRITERIA DECISION MAKING VERSUS FUZZY EXPERT SYSTEMS

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Abstract
Brazil is the largest country of Latin America and the fifth in the world, by geographical area or by population. Due to the dimensions of the country, Brazilian real estate market presents a wide variety of economic conditions. The market is warming with the coming of worldwide sporting events, mainly in Rio de Janeiro. This paper presents results for the evaluations of residential properties when two different methods are applied. In a typical mid-size Brazilian city, real data were collected. At first, TODIM, a multicriteria decision method founded on prospect theory that has appeared in the published literature since the early nineties and today of a widespread use, was applied. Then, a fuzzy expert system was also applied. In both applications, the same sets of alternatives and criteria were adopted. Although the results obtained with both applications are different, they are compatible.

Keywords: Brazil; fuzzy expert systems; multicriteria decision making; real estate market; TODIM method.

Resumo
O Brasil é, por área geográfica ou por população, o maior país da América Latina e o quinto no mundo. O mercado imobiliário brasileiro apresenta uma ampla gama de condições econômicas. Tal mercado esta aquecido, devido aos eventos desportivos internacionais, notadamente na Cidade do Rio de Janeiro. Este artigo mostra resultados das avaliações de propriedades residenciais obtidas por meio de dois métodos distintos aplicados a uma cidade brasileira de porte médio, através da coleta e da análise de dados reais. Primeiramente, fez-se uso do método multicritério TODIM, que tem como base a teoria dos prospectos, surgido na literatura internacional ao início dos anos noventa e hoje amplamente disseminado. Em seguida, empregou-se um sistema expert fundamentado na lógica nebulosa. Em ambas as aplicações empregaram-se os mesmos conjuntos de critérios e de alternativas. Os resultados foram diferentes, mas comparáveis, o que conduziu à uma análise visando futuras aplicações dos mesmos.

Palavras Chaves: Brasil; sistemas nebulosos; apoio multicritério à decisão; mercado imobiliário; método TODIM.

1. INTRODUCTION

Brazil is the largest country of Latin America and the fifth in the world, by geographical area or by population. Due to the dimensions of the country, Brazilian real estate market presents a wide variety of economic conditions. The market is warming with the coming of worldwide sporting events, mainly in Rio de Janeiro: FIFA World Cup, in 2014, and Summer Olympics, in 2016.

The appraisal of Brazilian residential properties is regulated by Brazilian National Standards Organization (Associação Brasileira de Normas Técnicas, 2011), the normative body responsible for technical standards in Brazil. Brazilian standards allow the use of different techniques to the evaluation of a property including comparative or formulated approaches. To deal with differences between the subject property and the comparable properties, several intelligent techniques, as artificial neural networks (ANN), data envelopment analysis (DEA) and fuzzy expert system, to name a few, have been applied (Lins, Novaes, & Legey, 2005). TODIM method (acronym in Portuguese for Interactive Multicriteria Decision Making), was previously applied to evaluate residential properties available for rent at Volta Redonda, a typical midsize city (200,000 inhabitants) from Rio de Janeiro (Gomes & Rangel, 2009). This paper presents the application of a fuzzy expert system to the same data. Although the results obtained with both applications are different, they are compatible.

2. THEORY BACKGROUND

2.1. INTERACTIVE MULTICRITERIA DECISION MAKING

TODIM (Gomes & Lima, 1992) is a discrete multicriteria method based on Prospect Theory (Kahneman & Tversky, 1979). TODIM has similarities with other multicriteria methods as ELECTRE (Roy & Bouyssou, 1993) and PROMETHEE (Barba-Romero & Pomerol, 2000). However, while practically all other multicriteria methods start from the premise that the decision maker always looks for some maximum overall value, TODIM method makes use of a measurement of overall value calculable according to Prospect Theory.

TODIM application requires numerical values for the evaluation of the alternatives regarding the criteria. For qualitative criteria, alternatives can be evaluated in a verbal scale, but it must be then transformed into a cardinal scale. The numerical evaluation for the alternatives regarding to all the criteria composes the matrix of evaluation. This matrix must be normalized, for each criterion: the value for one alternative must be divided by the sum of values for all the alternatives. This way, a stochastic matrix is obtained, that is, a matrix where all the components are in-between zero to one, and every column sums equal to one. This is the matrix of normalized alternatives' scores against criteria, $\mathrm{P} = p_{nm}$, with n indicating the number of alternatives and m the number of criteria.

The next step is the attribution of weights for the criteria. Usually, weights are attributed by decision makers using a linear 1 to 5 scale, similar to the Likert scale (Likert, 1932). The decision makers must indicate a criterion r as the reference criterion. The criterion with the highest weight is usually chosen. The vector of weights, $\mathrm{w}_{r} = w_{rc}$, is composed by the weight of the criterion c divided by the weight of the reference criterion r.

The measurement of dominance $\delta(A_i, A_j)$ of each alternative A_i over each alternative A_j, incorporate concepts of Prospect Theory, according to Equation 1 to 4.

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j), \forall(i, j) \quad (1)$$

where:

$$\Phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(P_{ic} - P_{jc})}{\sum_{c=1}^{m} w_{rc}}} & se \quad (P_{ic} - P_{jc}) > 0, \quad (2) \\ 0 & se \quad (P_{ic} - P_{jc}) = 0, \quad (3) \\ \dfrac{-1}{\theta}\sqrt{\dfrac{(\sum_{c=1}^{m} w_{rc})\ (P_{jc} - P_{ic})}{w_{rc}}} & se \quad (P_{ic} - P_{jc}) < 0, \quad (4) \end{cases}$$

The expression Φ_c (A_i, A_j) is the contribution of criterion *c* to the dominance of alternative A_i over alternative A_j. If p_{ic} was greater than p_{jc}, it will represent a gain for δ(A_i, A_j); if p_{ic} and p_{jc} were equal, then a zero will assigned to δ(A_i, A_j); if p_{ic} was less than p_{jc}, then Φ_c(A_i, A_j) will be a loss to δ(A_i, A_j).

The function Φ_c(A_i, A_j) allows the adjustment of problem data to the Prospect Theory, that is, considering the aversion to risk and the propensity to risk. This function has the shape of an ''S'', as presented in Figure 1. Above the horizontal axis, that is for value equal to zero, there is a concave curve representing the gains; below the horizontal axis, there is a convex curve representing the losses. The concave part reflects the aversion to risk in the face of gains and the convex part, in turn, symbolizes the propensity to risk when dealing with losses. θ is the attenuation factor of the losses. Different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant.

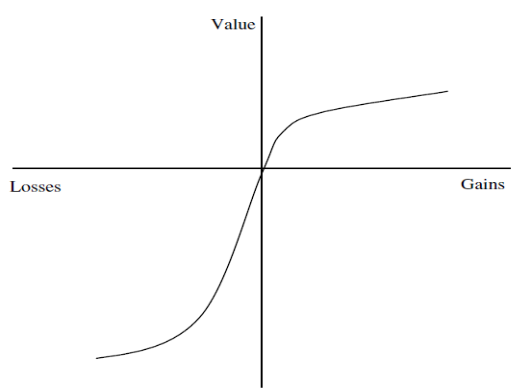

Figure 1 Value function of the TODIM method (Gomes & Lima, 1992)

The overall value for alternative Ai, ξi, is obtained with the idealized sum presented in Equation 5.

$$\xi_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}{\max \sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)} \quad (5)$$

2.2. FUZZY EXPERT SYSTEM

Fuzzy Sets Theory (Zadeh, 1965) is quite used in artificial intelligence to model the knowledge about a certain domain of expertise (Valls et al., 2010). In classical sets theory, elements either belong to a set, or not. In Fuzzy Sets Theory, elements can belong to a set with a certain degree (Bobillo, Delgado, & Gómez-Romero, 2009). This degree is more formally referred as membership. Every fuzzy set, A, is characterized by a membership function, $f_A(x)$, which associates every element, x, to a real number in the interval [0, 1]. As in classical theory, $f_A(x) = 0$ means no-membership, and $f_A(x) = 1$ means full-membership.

The establishment of a membership functions is a process referred as fuzzification (Seising, 2007). Because of its simplicity and easy computation, triangular membership functions are commonly used in practice. As presented in Figure 2, a triangular fuzzy set (TFS) has a triangular membership function. A TFS is usually represented as a vector, (α, β, γ), where $f_A(\alpha) = f_A(\gamma) = 0$, and $f_A(\beta) = 1$.

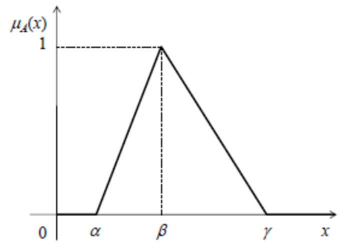

Figure 2 Triangular fuzzy set

An expert system consists of a database of facts and a database of rules. One of the most popular expert systems is the Mamdani Model (Bobillo, Delgado, & Gómez-Romero, 2009). In a Mamdani Model there are if-then rules of the type: if A then B, where all A and B are fuzzy propositions. These propositions must be established by experts. For every clause in the rule, the matching degree between the current value for the variable and a linguistic label must be computed. The clauses must be aggregated, using the Minimum Fuzzy Operator. If more than one rule implies in the same result, the rules must be aggregated, using the Maximum Fuzzy Operator. The overall matching degree can be obtained, also using the Minimum Fuzzy Operator. This degree is referred as alpha-cut, or α-cut (Bertoluzza, Solcia, & Capodieci, 2001). The α -cut level will generate a new Fuzzy Set, with a trapezoidal membership function, as presented in Figure 3. A real number may be obtained from the centroid of gravity (COG) of the resulting fuzzy set, within a process referred as defuzzification.

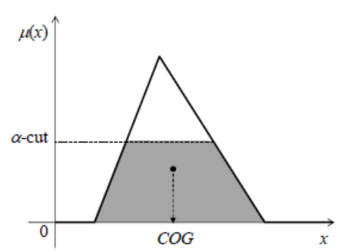

Figure 3 Defuzzification by the centroid of gravity

3. EVALUATION OF RESIDENTIAL PROPERTIES

3.1. DATA COLLECTION

Volta Redonda is a city situated in the South of the State of Rio de Janeiro, Brazil. It has approximately 260,000 inhabitants. There are a large number of properties, residential and commercial, rented or available for rent. The major steel plant installed in the city in the 1940's is a landmark of Brazilian industrialization. Because of this industrial vocation, Volta Redonda was nicknamed Steel City. However, its economy is quite diverse on services as education and transportation, to name a few.

Based on interviews with local real estate agents, eight criteria were selected for this particular application: localization (C_1), constructed area (C_2), construction quality (C_3), state of conservation (C_4), garage spaces (C_5), rooms (C_6), attractions (C_7), and security (C_8). Criteria C_2, C_5, C_6 and C_8, were quantitative. The constructed area was measured by m^2. Security was more than quantitative, a crisp criterion, that is, a property has security or has not. There were no middle points in the evaluation of alternatives for this criterion. Tables 1 to 4 present the possible scores to evaluation of alternatives according to the qualitative criteria.

Table 1 Possible scores to evaluation of localization

Localization	Score
Periphery	1
Between periphery and an average location	2
Average location	3
Good location	4
Excellent location	5

Table 2 Possible scores to evaluation of construction quality

Construction quality	Score
Low standard	1
Average standard	2
High standard	3

Table 3 Possible scores to evaluation of state of conservation

State of conservation	Score
Bad	1
Average	2
Good	3
Very good	4

Table 4 Possible scores to evaluation of attractions

Attractions	Score
Without attractions	0
Backyard or terrace	1
Barbecue	2
Swimming pool	3
Swimming pool, barbecue and others	4

Weights from 1 to 5 were assigned to the criteria, where 1 was assigned to criterion considered as of lowest importance and 5 to criteria of highest importance. Location (C_1) was indicated as the reference criterion. Table 5 presents the assigned weighted and the vector of weights, $w_1 = w_{1c}$.

Table 5 Weights of criteria

Criterion	Assigned weight	Vector of weights
Localization	5	1
Constructed area	3	0.6
Quality of construction	2	0.4
State of conservation	4	0.8
Garage spaces	1	0.2
Rooms	2	0.4
Attractions	1	0.2
Security	2	0.4

Fifteen properties in different neighbourhoods of Volta Redonda were evaluated. The identification of properties and locations was not in the purposes of this paper. Then the alternatives were simply named as A_1 to A_{15}. Table 6 presents the scores assigned to the alternatives according to the qualitative criteria (C_1, C_3, C_4 and C_7) and the real data for the quantitative criteria (C_2, C_5, C_6 and C_8). Table 7 presents the normalized score for the alternatives against the criteria.

Table 6 Evaluation of alternatives against the criteria

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	3	290	3	3	1	6	4	0
A_2	4	180	2	2	1	4	2	0
A_3	3	347	1	2	2	5	1	0
A_4	3	124	2	3	2	5	4	0
A_5	5	360	3	4	4	9	1	1
A_6	2	89	2	3	1	5	1	0
A_7	1	85	1	1	1	4	0	1
A_8	5	80	2	3	1	6	0	1
A_9	2	121	2	3	0	6	0	0
A_{10}	2	120	1	3	1	5	1	0
A_{11}	4	280	2	2	2	7	3	1
A_{12}	1	90	1	1	1	5	2	0
A_{13}	2	160	3	3	2	6	1	1
A_{14}	3	320	3	3	2	8	2	1
A_{15}	4	180	2	4	1	6	1	1

Table 7 Normalized score for the alternatives against the criteria

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.068	0.103	0.100	0.075	0.045	0.069	0.174	0
A_2	0.091	0.064	0.067	0.050	0.045	0.046	0.087	0
A_3	0.068	0.123	0.033	0.050	0.091	0.057	0.043	0
A_4	0.068	0.044	0.067	0.075	0.091	0.057	0.174	0
A_5	0.114	0.127	0.100	0.100	0.182	0.103	0.043	0.143

A_6	0.045	0.031	0.067	0.075	0.045	0.057	0.043	0
A_7	0.023	0.030	0.033	0.025	0.045	0.046	0	0.143
A_8	0.114	0.028	0.067	0.075	0.045	0.069	0	0.143
A_9	0.045	0.043	0.067	0.075	0	0.069	0	0
A_{10}	0.045	0.042	0.033	0.075	0.045	0.057	0.043	0
A_{11}	0.091	0.099	0.067	0.050	0.091	0.080	0.130	0.143
A_{12}	0.023	0.032	0.033	0.025	0.045	0.057	0.087	0
A_{13}	0.045	0.057	0.100	0.075	0.091	0.069	0.043	0.143
A_{14}	0.068	0.113	0.100	0.075	0.091	0.092	0.087	0.143
A_{15}	0.091	0.064	0.067	0.100	0.045	0.069	0.043	0.143

The overall values presented in Table 8 can be obtained simply aggregating the normalized scores for the alternatives (Table 7), multiplied by the vectors of normalized weights (Table 5). The overall values were also idealized to allow a better comparison with the TODIM application presented in Section 3.1. The bolded A_5, A_{11}, and A_{14} have the three highest overall values; A_{12} have the lowest value.

Table 8 Overall values for the alternatives without Prospect Theory

Alternative	Overall value	Idealized overall value
A_1	0.075	0.664
A_2	0.060	0.531
A_3	0.061	0.540
A_4	0.064	0.567
A_5	0.113	1.000
A_6	0.048	0.423
A_7	0.040	0.350
A_8	0.078	0.686
A_9	0.046	0.409
A_{10}	0.046	0.408
A_{11}	0.088	0.773
A_{12}	0.031	0.275
A_{13}	0.073	0.642
A_{14}	0.091	0.806
A_{15}	0.085	0.746

3.2. TODIM APPLICATION

As in many other TODIM applications (Ribeiro, Passos, & Teixeira, 2012), $\theta = 1$ was adopted. To illustrate TODIM computation, let us consider the pair A_1 and A_2:

- For C_1, $p_{11} < p_{12}$, then $\Phi_1 = -\frac{1}{1}\sqrt{\frac{4(0.091-0.068)}{1}} \cong -0.303$
- For C_2, $p_{12} > p_{22}$, then $\Phi_2 = \sqrt{\frac{0.6(0.103-0.054)}{4}} \cong 0.076$
- For C_3, $p_{13} > p_{23}$, then $\Phi_3 = \sqrt{\frac{0.4(0.100-0.057)}{4}} \cong 0.057$

- For C_4, $p_{14} > p_{24}$, then $\Phi_4 = \sqrt{\frac{0.8(0.075-0.050)}{4}} \cong 0.071$
- For C_5, $p_{15} = p_{15}$, then $\Phi_5 = 0$
- For C_6, $p_{16} > p_{26}$, then $\Phi_6 = \sqrt{\frac{0.4(0.069-0.046)}{4}} \cong 0.048$
- For C_7, $p_{17} > p_{27}$, then $\Phi_7 = \sqrt{\frac{0.05(0.174-0.087)}{4}} \cong 0.066$
- For C_8, $p_{11} = p_{12}$, then $\Phi_8 = 0$

Then, substituting values in Equation 1, $\delta(A_1, A_2) \cong 0.017$. In analogy, $\delta(A_1, A_3) \cong$ -1.05; $\delta(A_1, A_4) \cong$ -0.768; $\delta(A_1, A_5) \cong$ -4.54; $\delta(A_1, A_6) \cong$ = 0.351; $\delta(A_1, A_7) \cong$ -0.661; $\delta(A_1, A_8) \cong$ -1.365; $\delta(A_1, A_9) \cong$ 0.368; $\delta(A_1, A_{10}) \cong$ 0.367; $\delta(A_1, A_{11}) \cong$ -2.59; $\delta(A_1, A_{12}) \cong$ 0.491; $\delta(A_1, A_{13}) \cong$ = -1.91; $\delta(A_1, A_{14}) \cong$ -2.82; and $\delta(A_1, A_{15}) \cong$ -1.64. Adding values, $\sum_{j=1}^{n} \delta(A_1, A_j)$ = -15.76. The minimum and maximum sums are $\sum_{j=1}^{n} \delta(A_{12}, A_j)$ = -44.23 and $\sum_{j=1}^{n} \delta(A_5, A_j)$ = 0.343. By substituting in Equation 2, one gets $\xi_1 = \frac{-15.75-44.23}{0.343-44.23} \cong 0.644$.

Table 9 presents he overall values for the alternatives with Prospect Theory. These overall values were obtained with the ξ_i.

Table 9 Overall values for the alternatives with Prospect Theory

Alternative	Overall value
A_1	0.692
A_2	0.386
A_3	0.399
A_4	0.620
A_5	1
A_6	0.286
A_7	0
A_8	0.441
A_9	0.020
A_{10}	0.213
A_{11}	0.858
A_{12}	0.107
A_{13}	0.719
A_{14}	0.937
A_{15}	0.673

It can be noted in Table 8 and Table 9 that the use of Prospect Theory implies in a different rank. However, the top three (the bolded A_5, A_{11} and A_{14}) and the bottom four (A_7, A_9, A_{10} and A_{12}) will be the same.

3.3. EXPERT SYSTEM APPLICATION

A Mamdani Model based fuzzy expert system was developed, as presented in Figure 4. To facilitate the implementation of this system the fuzzyTECH software (INFORM GmbH, 2012) was selected, mainly because of it has been successfully applied in practice (Valls *et al*., 2010).

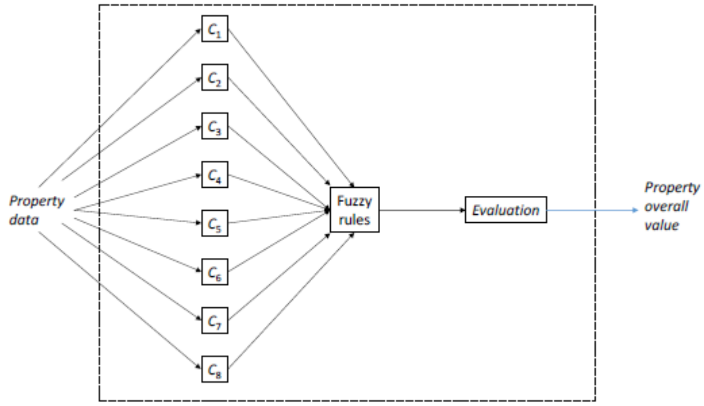

Figure 4 Expert system for evaluation of residential properties

In fuzzyTECH, three Triangle fuzzy sets were defined for each qualitative criteria (C_1, C_3, C_4 and C_7): *bad*, *average* and *good*. For the quantitative criteria (C_2, C_5, C_6 and C_8), only one set was defined: *good*. For the evaluation two sets were defined: *bad* and *good*. Figures 5 to 7 present the fuzzy sets for *location*, the fuzzy set for *constructed area*, and the fuzzy sets for *evaluation*.

The expert system is accomplished in $3^4 = 81$ rules. In fuzzyTECH, these rules were created in groups of three rules, inserted in blocks of three groups. Table 10 presents the first and the last groups. When there is, at least, one *bad* set for a qualitative criterion the production is a *bad* evaluation. Table 11 presents the overall value obtained with defuzzification by COG.

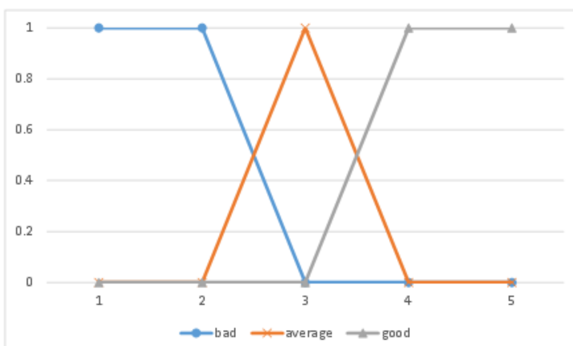

Figure 5 Fuzzy sets for location

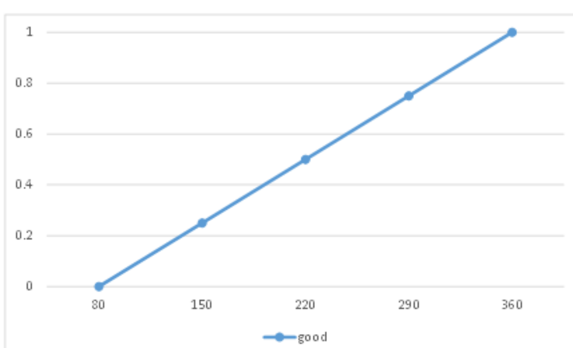

Figure 6 Fuzzy set for constructed area

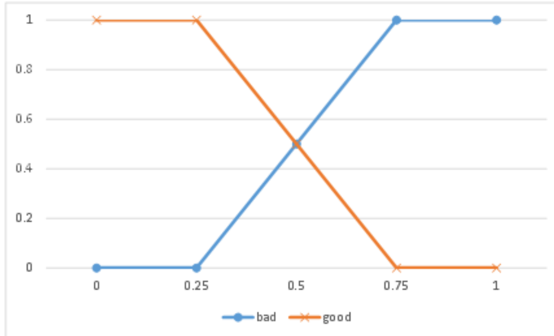

Figure 7 Fuzzy sets for evaluation

Table 10 Fuzzy rules

Rule	Input				Output
	Location	Construction quality	State of conservation	Attractions	Evaluation
1	Bad	Bad	Bad	Bad	Bad
2	Bad	Bad	Bad	Average	Bad
3	Bad	Bad	Bad	Good	Bad
…	…	…	…	…	…
79	Good	Good	Good	Bad	Bad
80	Good	Good	Good	Average	Good
81	Good	Good	Good	Good	Good

Table 11 Overall values for the alternatives with fuzzy expert system

Alternative	Overall value
A_1	0
A_2	0
A_3	0
A_4	0
A_5	0.259
A_6	0
A_7	0
A_8	0
A_9	0
A_{10}	0
A_{11}	0.452
A_{12}	0
A_{13}	0.259
A_{14}	0.741
A_{15}	0.259

The first observation from Table 11 is that for 10 of 15 alternatives, the evaluation resulted in overall values equal to zero. This is due to the fuzzy expert system design. Alternatives with the lowest value according to a quantitative criterion received a α-cut equal to zero. This situation could be avoided adopting an origin lower than the lowest value presented in

Table **6**. But it will be only possible for C_2, C_5 and C_6, and not for C_8, as this last one is a crisp criterion. This way, A_1, A_2, A_3, A_4, A_6, A_9, A_{10}, and A_{12} will still zero overall value alternatives. Nevertheless, it seems to be plausible, since they are residential properties without security system. I.e., security is a major issue in Brazil, particularly in Rio de Janeiro.

Another observation regarding results with fuzzy expert system application is the top three alternatives with TODIM application (bolded in Table 11) have non zero overall values.

Summarizing, both applications resulted A_5, A_{11}, and A_{14} as top alternatives, and A_7, A_9, A_{10} and A_{12} in the bottom. Therefore, besides the differences in the overall values, the results from both applications can be considered as compatible each other.

4. DISCUSSION

In this paper, evaluations of residential properties with two different methods are presented. The same set of criteria and alternatives were utilized in both applications. TODIM, a multicriteria decision aiding method based on Prospect Theory and initially conceived in Brazil in the early nineties, was applied. The main contribution of this paper is the application of fuzzy expert systems and its comparison with a multicriteria method.

As presented in Section 3, TODIM application was entirely conducted with electronic spreadsheets. Unlike this, fuzzy expert system application required a proper software, which demands specific knowledge and skills. In both applications, Sensitivity Analyses were conducted and did not affect the results.

As in most other multicriteria methods, with TODIM is possible to consider different weight for the criteria. Unlike this, the fuzzy expert system application considered the same weight for every criterion. Even it is possible to consider different weights for the criteria (named as variables) in a fuzzy expert system, it is not as simple as in a multicriteria method application. Perhaps, using the same weight for different criteria is a default approach in fuzzy expert system applications. Surprisingly, this divergence does not implied in major differences in the results.

The main topic for future research emerging from our findings is comparison between TODIM and other intelligent techniques (as ANN, or DEA). Similar comparison also can be done with fuzzy expert system and other multicriteria decision making methods.

Data were collected in a typical Brazilian midsize city. However, it can be extrapolated to other situations, mutatis mutandis. So far, the most important contribution of this work is to show how TODIM and fuzzy expert systems can be useful tools aiding a subjective multicriteria problem of evaluate residential properties.

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